MAS156: Mathematics (Electrical and Aerospace)

Semester Year, 2011/12; 20 credits

Lecturer: Semester 1: Dr Sam Marsh (S.J.Marsh@shef, Room G15 Hicks Building, Ext 23792)
Semester 2: Dr Glenn Vickers (tbc)

Home page: `Semester 1: http://sam-marsh.staff.shef.ac.uk/mas156`
(or via MOLE2)

Outline syllabus.

Semester 1

- Functions of a real variable
- Complex Numbers
- Differentiation
- Partial Differentiation
- Vectors

Semester 2

- Integration
- Ordinary Differential Equations
- Taylor and Maclaurin Series
- L'Hôpital's Rule
- Equation Solving
- Matrix Algebra.

Aims. This year-long module aims to help to develop the mathematical skills necessary to support engineering studies and to provide students with the appropriate foundations for further mathematical studies.

Learning outcomes.

- Sketch simple functions
- Understand of complex numbers
- Be familiar with the techniques of differentiation and aware of applications
- Be able to manipulate vectors.
- Understand techniques and uses of integration
- Solve ordinary differential equations of the variables separable and constant coefficient types, including the use of the Laplace transform, and be aware of elementary applications of o.d.e.s,
- Be familiar with the elements of convergence of series, Maclaurin series, limits and equation solving
- Be familiar with the techniques of matrix algebra
- Solve systems of linear simultaneous equations.

Teaching methods. Lectures, tutorials, problem solving
40 lectures, 20 tutorials, 0 practicals.

Assessment. One formal 3 hour written examination at the end of the course (80%) plus one piece of coursework towards the end of Semester 1 (20%).

Full syllabus.

Semester 1.

Number Systems and Set Theory (Handout). Types of real numbers - integers, rational, irrational. Laws of addition, multiplication and division. Set theory notation.

Functions of a Real Variable. Idea of a function. (Even and odd functions, periodic functions). Inverse functions. Formula manipulation. Multivaluedness. Graphs (lin/lin, ln/lin, ln/ln). Transformations using graphs: $f(t)$, $f(-t)$, $f(t-a)$, $f(at)$. Exponential. Logarithm. a^x. Definitions and properties. Hyperbolic functions. Graphs.

Complex Numbers. Notion of a complex number. Real and imaginary parts. Complex conjugate. Addition, subtraction, multiplication and division. Argand diagram - modulus, argument. Principal argument. De Moivre's theorem. n'th roots of complex numbers. Euler's relation, e^z manipulation. e^z where $z = f(t)$, e^{jt}, amplitude, phase.

Differentiation. Introduction to limits. Continuity (informal). Derivative. Rules for differentiation (products, quotients, function of a function, inverse function). Maxima and minima, greatest and least values, points of inflection. Graphs using derivatives, limits, etc.

Partial Differentiation. Functions of more than 1 variable, Partial Differentiation Equation of a surface. Definition of partial differentiation as a limit. Higher order derivatives. Simple examples of finding $\partial f/\partial x$, $\partial f/\partial y$, $\partial^2 f/\partial x^2$, $\partial^2 f/\partial x \partial y$, $\partial^2 f/\partial y^2$.

Vectors. Definition of a vector. Magnitude. Addition and subtraction. Laws of vector algebra (with proofs). Unit vector. The base vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$. Components. Resolved form of a vector. Scalar product $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|cos\theta$. Interpretation as projection. Laws of scalar products. Component form of scalar product. Vector product $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|sin\theta\ \ \mathbf{n}$. Laws of vector products. Component form of vector product. Differentiation of a vector. Rules for differentiation. Vector equation of line.

Semester 2.

Integration (6 lectures). Indefinite integrals of simple functions e.g. x^n, e^{ax}, $sin(ax)$, $cos(ax)$, $sinh(ax)$, $cosh(ax)$, $sinh^2(ax)$, $cosh^2(ax)$. Definite Integral. Properties of a definite integral. Evaluation of definite integrals using indefinite integral. Integration as area, use of geometry, even and odd functions. Methods of integration (substitution, integration by parts). Mention of improper integrals. Integration of piecewise differentiable/continuous functions especially functions with zero ranges.

Ordinary Differential Equations (7 lectures). Notion of a differential equation. Order. First order equations: variables separable $dy/dt = f(t)g(y)$, manipulating d.e. into this form. First and second order equations with constant coefficients: method of finding the complementary functions, methods of finding particular integrals, undetermined coefficients (Briefly). Laplace transforms. Applications of d.e.: 1st order equation for electrical circuit with inductance and resistance. Forced oscillation and response applied to electrical and mechanical systems pointing out mathematical analogy. Taylor and Maclaurin series, L'Hôpital's Rule.

Equation Solving (4 lectures). Notion of convergence. Taylor and Maclaurin series. L'Hôpital's Rule (deduced from Taylor's theorem). Graphical solution of equations. Iterative equation solving (Newton-Raphson).

Matrix Algebra (6 lectures). Definition of an $m \times n$ matrix $A = [a_{ij}]$ or $A = (a_{ij})$. Matrix as an operator. Square matrix. Diagonal elements. Trace. Zero matrix. Equality of matrices. Definition and laws of addition, subtraction and multiplication by a number. Transpose AT. Matrix multiplication and properties of. Definition of second and third order determinants. Definition of inverse of a matrix. $(AB)^{-1} = B^{-1}A^{-1}$. Non-homogeneous and homogeneous systems of equations. Numerical solution methods.

Recommended books.

- **B** Croft, Davison and Hargreaves "Engineering Mathematics: a foundation for electronic, electrical, communications and systems engineer" (Shelfmark 510.2462 (C), ISBN 0130268585)
- **B** Stroud "Engineering Mathematics" (Shelfmark 510.2462 (S), ISBN 9781403942463)

(**A** = essential, **B** = recommended, **C** = background.)